

### Lesson Practice 23A

1. Let  $u = 2 + X$ ;  $\frac{du}{dX} = 1$ ;  $du = dX$

$$\begin{aligned} \int (2 + X)^4 dX &= \int u^4 du \\ &= \frac{u^5}{5} + C \\ &= \frac{1}{5}(2 + X)^5 + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{5}(2 + X)^5 \right) &= \frac{1}{5} \cdot 5(2 + X)^4 \\ &= (2 + X)^4 \end{aligned}$$

2. Let  $u = t + 1$ ;  $\frac{du}{dt} = 1$

$$\begin{aligned} \int \frac{dt}{\sqrt{t+1}} &= \int \frac{du}{\sqrt{u}} \\ &= \int u^{-\frac{1}{2}} du \\ &= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 2\sqrt{u} + C \\ &= 2\sqrt{t+1} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dt} (2\sqrt{t+1}) &= 2 \cdot \frac{1}{2} (t+1)^{-\frac{1}{2}} \\ &= \frac{1}{\sqrt{t+1}} \end{aligned}$$

3. Let  $u = 3 + X^2$ ;  $\frac{du}{dX} = 2X$ ;  $du = 2XdX$

$$\begin{aligned} \int (3 + X^2)^{10} 2XdX &= \int u^{10} du \\ &= \frac{u^{11}}{11} + C \\ &= \frac{1}{11}(3 + X^2)^{11} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{11}(3 + X^2)^{11} \right) &= \frac{1}{11} \cdot 11(3 + X^2)^{10} \cdot 2X \\ &= 2X(3 + X^2)^{10} \end{aligned}$$

4. Let  $u = 4X$ ;  $\frac{du}{dX} = 4$ ;  $du = 4dX$

$$\begin{aligned} \int \csc^2(4X) \cdot 4dX &= \int \csc^2(u) du \\ &= -\cot(u) + C \\ &= -\cot(4X) + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} (-\cot(4X)) &= -(-\csc^2(4X) \cdot 4) \\ &= 4\csc^2(4X) \end{aligned}$$

5. Let  $u = \sin(3\theta)$ ;  $\frac{du}{d\theta} = \cos(3\theta) \cdot 3$ ;  
 $du = \cos(3\theta) \cdot 3(d\theta)$

$$\begin{aligned} \int \sin^3(3\theta) \cos(3\theta) d\theta &= \frac{1}{3} \int \sin^3(3\theta) \cos(3\theta) d\theta \cdot 3 \\ &= \frac{1}{3} \int u^3 du \\ &= \frac{1}{3} \frac{u^4}{4} + C \\ &= \frac{1}{12} u^4 + C \\ &= \frac{1}{12} (\sin(3\theta))^4 + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{d\theta} \left( \frac{1}{12} \sin(3\theta)^4 \right) &= \frac{1}{12} \cdot 4(\sin(3\theta))^3 \cdot \cos(3\theta) \cdot 3 \\ &= (\sin(3\theta))^3 \cos(3\theta) \end{aligned}$$

6. Let  $u = a - Y$ ;  $\frac{du}{dY} = -1$ ;  $du = -dY$

$$\begin{aligned} \int \frac{dY}{(a - Y)^2} &= - \int \frac{-dY}{(a - Y)^2} \\ &= - \int \frac{du}{u^2} \\ &= - \int u^{-2} du \\ &= - \frac{u^{-1}}{-1} + C \\ &= \frac{1}{a - Y} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dY} \left( \frac{1}{a - Y} \right) &= \frac{d}{dY} (a - Y)^{-1} \\ &= -(a - Y)^{-2} (-1) \\ &= \frac{1}{(a - Y)^2} \end{aligned}$$

7. Let  $u = 4X$ ;  $\frac{du}{dX} = 4$ ;  $du = 4dX$

$$\begin{aligned} \int e^{4X} dX &= \frac{1}{4} \int e^{4X} dX \cdot 4 \\ &= \frac{1}{4} \int e^u du \\ &= \frac{1}{4} e^u + C \\ &= \frac{1}{4} e^{4X} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{4} e^{4X} \right) &= \frac{1}{4} e^{4X} \cdot 4 \\ &= e^{4X} \end{aligned}$$

8. Let  $u = 1 + e^X$ ;  $\frac{du}{dX} = e^X$ ;  $du = e^X dX$

$$\begin{aligned} \int e^X (1 + e^X)^8 dX &= \int u^8 du \\ &= \frac{u^9}{9} + C \\ &= \frac{1}{9} (1 + e^X)^9 + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left[ \left( \frac{1}{9} \cdot 1 + e^X \right)^9 \right] &= \frac{1}{9} \cdot 9(1 + e^X)^8 \cdot e^X \\ &= e^X (1 + e^X)^8 \end{aligned}$$

9. Let  $u = \ln(X)$ ;  $\frac{du}{dX} = \frac{1}{X}$ ;  $du = \frac{dX}{X}$

$$\begin{aligned} \int \frac{\ln(X)^2}{X} dX &= \int u^2 du \\ &= \frac{u^3}{3} + C \\ &= \frac{1}{3} (\ln(X))^3 + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{3} (\ln(X))^3 \right) &= \frac{1}{3} \cdot 3 (\ln(X))^2 \cdot \frac{1}{X} \\ &= \frac{\ln^2(X)}{X} \end{aligned}$$

10. Let  $u = 2t + 1$ ;  $\frac{du}{dt} = 2$ ;  $du = 2dt$

$$\begin{aligned} \int \frac{3dt}{\sqrt{2t+1}} &= \frac{1}{2} \cdot 3 \int \frac{dt}{\sqrt{2t+1}} \cdot 2 \\ &= \frac{3}{2} \int \frac{du}{\sqrt{u}} \\ &= \frac{3}{2} \int u^{-\frac{1}{2}} du \\ &= \frac{3}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 3\sqrt{u} + C \\ &= 3\sqrt{2t+1} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dt} (3\sqrt{2t+1}) &= 3 \cdot \frac{1}{2} (2t+1)^{-\frac{1}{2}} \cdot 2 \\ &= \frac{3}{\sqrt{2t+1}} \end{aligned}$$

### Lesson Practice 23B

1. Let  $u = X - 3$ ;  $\frac{dY}{dX} = 1$

$$\begin{aligned} \int (X-3)^{22} dX &= \int u^{22} du \\ &= \frac{u^{23}}{23} + C \\ &= \frac{1}{23} (X-3)^{23} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{23} (X-3)^{23} \right) &= \frac{1}{23} \cdot 23 (X-3)^{22} \\ &= (X-3)^{22} \end{aligned}$$

2. Let  $u = 2X^2 + 1$ ;  $\frac{du}{dX} = 4X$ ;  $du = 4XdX$

$$\begin{aligned} \frac{1}{4} \int X\sqrt{2X^2+1} dX \cdot 4 &= \frac{1}{4} \int \sqrt{u} du \\ &= \frac{1}{4} \int u^{\frac{1}{2}} du \\ &= \frac{1}{4} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{1}{6} (2X^2+1)^{\frac{3}{2}} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{6} (2X^2+1)^{\frac{3}{2}} \right) &= \frac{1}{6} \cdot \frac{3}{2} (2X^2+1)^{\frac{1}{2}} \cdot 4X \\ &= X\sqrt{2X^2+1} \end{aligned}$$

3. Let  $u = 1 - X^3$ ;  $\frac{du}{dX} = -3X^2$ ;  $du = -3X^2 dX$

$$\begin{aligned} -\frac{1}{3} \int \frac{X^2}{\sqrt{1-X^3}} dX(-3) &= -\frac{1}{3} \int \frac{du}{\sqrt{u}} \\ &= -\frac{1}{3} \int u^{-\frac{1}{2}} du \\ &= -\frac{1}{3} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= -\frac{2}{3} \sqrt{u} + C \\ &= -\frac{2}{3} \sqrt{1-X^3} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( -\frac{2}{3} \sqrt{1-X^3} \right) &= -\frac{2}{3} \cdot \frac{1}{2} (1-X^3)^{-\frac{1}{2}} (-3X^2) \\ &= \frac{X^2}{\sqrt{1-X^3}} \end{aligned}$$

4. Let  $u = 1 + X^3$ ;  $\frac{du}{dX} = 3X^2$ ;  $du = 3X^2 dX$

$$\begin{aligned} \int (1+X^3)^3 \cdot 3X^2 dX &= \int u^3 du \\ &= \frac{u^4}{4} + C \\ &= \frac{1}{4} (1+X^3)^4 + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{4} (1+X^3)^4 + C \right) &= \frac{1}{4} \cdot 4 (1+X^3)^3 (3X^2) \\ &= 3X^2 (1+X^3)^3 \end{aligned}$$

5. Let  $u = 2X - 3$ ;  $\frac{du}{dX} = 2$ ;  $du = 2dX$

$$\begin{aligned} \frac{1}{2} \int \cos(2X-3) dX \cdot 2 &= \frac{1}{2} \int \cos(u) (du) \\ &= \frac{1}{2} \sin(u) + C \\ &= \frac{1}{2} \sin(2X-3) + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{1}{2} \sin(2X-3) \right) &= \frac{1}{2} \cos(2X-3) \cdot 2 \\ &= \cos(2X-3) \end{aligned}$$

6. Let  $u = \sqrt{\theta} = \theta^{\frac{1}{2}}$ ;  $\frac{du}{d\theta} = \frac{1}{2} \theta^{-\frac{1}{2}}$ ;  $du = \frac{1}{2} \theta^{-\frac{1}{2}} d\theta$

$$\begin{aligned} 2 \int \frac{\cos(\sqrt{\theta} (d\theta)) \cdot \frac{1}{2}}{\sqrt{\theta}} &= 2 \int (\cos(u) (du)) \\ &= 2 \sin(u) + C \\ &= 2 \sin(\sqrt{\theta}) + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{d\theta} \left( 2 \sin(\sqrt{\theta}) \right) &= 2 \cos(\sqrt{\theta}) \cdot \frac{1}{2} \theta^{-\frac{1}{2}} \\ &= \frac{\cos(\sqrt{\theta})}{\sqrt{\theta}} \end{aligned}$$

7. Let  $u = \tan(\theta)$ ;  $\frac{du}{dX} = \sec^2(\theta)$ ;

$$du = \sec^2(\theta)(d\theta)$$

$$\begin{aligned} \int \sqrt{\tan(\theta)} \sec^2(\theta)(d\theta) &= \int u^{\frac{1}{2}} du \\ &= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{2}{3} u^{\frac{3}{2}} + C \\ &= \frac{2}{3} (\tan\theta)^{\frac{3}{2}} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{d\theta} \left( \frac{2}{3} (\tan(\theta))^{\frac{3}{2}} \right) &= \frac{2}{3} \cdot \frac{3}{2} (\tan(\theta))^{\frac{1}{2}} \sec^2(\theta) \\ &= \sqrt{\tan(\theta)} \sec^2(\theta) \end{aligned}$$

8. Let  $u = 2 - e^{2X}$ ;  $\frac{du}{dX} = -e^{2X} \cdot 2$ ;

$$du = -2e^{2X}dX$$

$$\begin{aligned} -\frac{1}{2} \int e^{2X}(2 - e^{2X})dX(-2) &= -\frac{1}{2} \int u(du) \\ &= -\frac{1}{2} \frac{u^2}{2} + C \\ &= -\frac{1}{4} u^2 + C \\ &= -\frac{1}{4} (2 - e^{2X})^2 + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{du} \left( -\frac{1}{4} (2 - e^{2X})^2 \right) &= -\frac{1}{4} \cdot 2(2 - e^{2X})(-e^{2X} \cdot 2) \\ &= e^{2X} \cdot (2 - e^{2X}) \end{aligned}$$

You could also multiply through which yields  $2e^{2X} - e^{4X}$  and integrate each term.

9. Let  $u = 1 - X$ ;  $\frac{du}{dX} = -1$ ;  $du = -dX$

$$\begin{aligned} -\int \frac{dX}{1-X}(-1) &= -\int \frac{du}{u} \\ &= -\ln(u) + C \\ &= -\ln(1-X) + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} (-\ln(1-X)) &= -\frac{1}{1-X}(-1) \\ &= \frac{1}{1-X} \end{aligned}$$

10. Let  $u = X^2 + 2X - 1$ ;  $\frac{du}{dX} = 2X + 2$ ;

$$du = (2X + 2)dX$$

$$\begin{aligned} \frac{1}{2} \int \frac{(X+1)dX}{\sqrt[3]{X^2 + 2X - 1}} \cdot 2 &= \frac{1}{2} \int \frac{du}{u^{\frac{1}{3}}} \\ &= \frac{1}{2} \int u^{-\frac{1}{3}} du \\ &= \frac{1}{2} \frac{u^{\frac{2}{3}}}{\frac{2}{3}} + C \\ &= \frac{3}{4} u^{\frac{2}{3}} + C \\ &= \frac{3}{4} (X^2 + 2X - 1)^{\frac{2}{3}} + C \end{aligned}$$

Check :

$$\begin{aligned} \frac{d}{dX} \left( \frac{3}{4} (X^2 + 2X - 1)^{\frac{2}{3}} \right) &= \\ &= \frac{3}{4} \cdot \frac{2}{3} (X^2 + 2X - 1)^{-\frac{1}{3}} (2X + 2) \\ &= \frac{X + 1}{\sqrt[3]{X^2 + 2X - 1}} \end{aligned}$$

## Lesson Practice 23C

1. Let  $u = t + 4$ ;  $\frac{du}{dt} = 1$ ;  $du = dt$

$$\begin{aligned}\int \frac{dt}{\sqrt{t+4}} &= \int \frac{du}{\sqrt{u}} \\ &= \int u^{-\frac{1}{2}} du \\ &= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{1}{2} \sqrt{u} + C \\ &= \frac{1}{2} \sqrt{t+4} + C\end{aligned}$$

Check :

$$\begin{aligned}\frac{d}{dt}(2\sqrt{t+4}) &= 2 \cdot \frac{1}{2}(t+4)^{-\frac{1}{2}} \\ &= \frac{1}{\sqrt{t+4}}\end{aligned}$$

2. Let  $u = 2 - 3Y$ ;  $\frac{du}{dY} = -3$ ;  $du = -3dY$

$$\begin{aligned}-\frac{1}{3} \int (2-3Y)^5 dY(-3) &= -\frac{1}{3} \int u^5(du) \\ &= -\frac{1}{3} \frac{u^6}{6} + C \\ &= -\frac{1}{18} u^6 + C \\ &= -\frac{1}{18} (2-3Y)^6 + C\end{aligned}$$

Check :

$$\begin{aligned}\frac{d}{dY}(-\frac{1}{18}(2-3Y)^6) &= -\frac{1}{18} \cdot 6(2-3Y)^5(-3) \\ &= (2-3Y)^5\end{aligned}$$

3. If you let  $u = X^2+1$ , then  $du = 2XdX$  which is not present. You must multiply the numerator and divide by the denominator before integrating.

$$\begin{aligned}\int \frac{(X^2+1)^2}{X^2} dX &= \int \frac{X^4+2X^2+1}{X^2} dX \\ &= \int X^2 dX + \int 2dX + \int X^{-2} dX \\ &= \frac{X^3}{3} + 2X + \frac{X^{-1}}{-1} + C \\ &= \frac{1}{3} X^3 + 2X - \frac{1}{X} + C\end{aligned}$$

Check :

$$\begin{aligned}\frac{d}{dX}(\frac{1}{3}X^3 + 2X - \frac{1}{X}) &= \frac{1}{3}(3X^2) + 2 - (-X^{-2}) \\ &= X^2 + 2 + X^{-2} \\ &= \frac{X^4 + 2X^2 + 1}{X^2} \\ &= \frac{(X^2+1)^2}{X^2}\end{aligned}$$

4. Let  $u = 1 + \tan(\theta)$ ;  $\frac{du}{d\theta} = \sec^2(\theta)$ ;  
 $du = \sec^2(\theta)(d\theta)$

$$\int \frac{\sec^2(\theta)(d\theta)}{(1 + \tan(\theta))^4} = \int \frac{du}{u^4} \int u^{-4} du$$

$$= \frac{u^{-3}}{-3} + C$$

$$= -\frac{1}{3}(1 + \tan(\theta))^{-3} + C$$

Check :

$$\frac{d}{d\theta} \left( -\frac{1}{3}(1 + \tan(\theta))^{-3} \right)$$

$$= -\frac{1}{3}(-3(1 + \tan(\theta))^{-4})\sec^2(\theta)$$

$$= \frac{\sec^2(\theta)}{(1 + \tan(\theta))^4}$$

5. Let  $u = 1 + \tan(\theta)$ ;  $\frac{du}{d\theta} = \sec^2(\theta)$ ;  
 $du = \sec^2(\theta)(d\theta)$

$$\int \frac{\sec^2(\theta)(d\theta)}{(1 + \tan(\theta))^4} = \int \frac{du}{u^4} \int u^{-4} du$$

$$= \frac{u^{-3}}{-3} + C$$

$$= -\frac{1}{3}(1 + \tan(\theta))^{-3} + C$$

Check :

$$\frac{d}{d\theta} \left( -\frac{1}{3}(1 + \tan(\theta))^{-3} \right)$$

$$= -\frac{1}{3}(-3(1 + \tan(\theta))^{-4})\sec^2(\theta)$$

$$= \frac{\sec^2(\theta)}{(1 + \tan(\theta))^4}$$

### Lesson Practice 23D

1. Let  $u = \cot(3\theta)$ ;  $\frac{du}{d\theta} = -\csc^2(3\theta) \cdot 3$ ;  
 $du = -3\csc^2(3\theta)(d\theta)$

$$-\frac{1}{3} \int \cot(3\theta) \csc^2(3\theta) d\theta (-3)$$

$$= -\frac{1}{3} \int u du$$

$$= -\frac{1}{3} \frac{u^2}{2} + C$$

$$= -\frac{1}{6} u^2 + C$$

$$= -\frac{1}{6} (\cot(3\theta))^2 + C$$

Check :

$$\frac{d}{d\theta} \left( -\frac{1}{6} (\cot(3\theta))^2 + C \right)$$

$$= -\frac{1}{6} \cdot 2(\cot(3\theta)) \cdot 3(-\csc^2(3\theta))$$

$$= \cot(3\theta) \csc^2(3\theta)$$

2. There is no substitution that works.  
 Multiply through and integrate.

$$\int (x^2 + 1)(x^2 - 2) dx = \int (x^4 - 2x^2 + x^2 - 2) dx$$

$$= \int (x^4 - x^2 - 2) dx$$

$$= \frac{x^5}{5} - \frac{x^3}{3} - 2x + C$$

$$= \frac{1}{5}x^5 - \frac{1}{3}x^3 - 2x + C$$

Check :

$$\frac{d}{dx} \left( \frac{1}{5}x^5 - \frac{1}{3}x^3 - 2x \right)$$

$$= \frac{1}{5} \cdot 5x^4 - \frac{1}{3} \cdot 3x^2 - 2$$

$$= x^4 - x^2 - 2$$

$$= (x^2 + 1)(x^2 - 2)$$

3. Let  $u = 2Y + 1$ ;  $\frac{du}{dY} = 2$ ;  $du = 2dY$

$$\begin{aligned}\int \frac{4dY}{(2Y+1)^3} &= 2 \int \frac{4dY \cdot \frac{1}{2}}{(2Y+1)^3} \\ &= 2 \int u^{-3} du \\ &= \frac{2u^{-2}}{-2} + C \\ &= -u^{-2} + C \\ &= -(2Y+1)^{-2} + C\end{aligned}$$

Check :

$$\begin{aligned}\frac{d}{dY} \left( -(2Y+1)^{-2} \right) &= - \left( -2(2Y+1)^{-3} (2) \right) \\ &= 4(2Y+1)^{-3}\end{aligned}$$

4. Let  $u = \tan(\theta) - 2$ ;  $\frac{du}{d\theta} = \sec^2(\theta)$ ;  
 $du = \sec^2(\theta)(d\theta)$

$$\begin{aligned}\int \frac{\sec^2(\theta)(d\theta)}{\sqrt{\tan(\theta) - 2}} &= \int u^{-\frac{1}{2}} du \\ &= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 2\sqrt{u} + C \\ &= 2\sqrt{\tan(\theta) - 2} + C\end{aligned}$$

Check :

$$\begin{aligned}\frac{d}{d\theta} \left( 2\sqrt{\tan(\theta) - 2} \right) &= 2 \cdot \frac{1}{2} (\tan(\theta) - 2)^{-\frac{1}{2}} \sec^2(\theta) \\ &= \sec^2(\theta) (\tan(\theta) - 2)^{-\frac{1}{2}}\end{aligned}$$

5. Let  $u = -2X^2$ ;  $\frac{du}{dX} = -4X$ ;  $du = -4XdX$

$$\begin{aligned}-\frac{1}{4} \int X e^{-2X^2} dX (-4) &= -\frac{1}{4} \int e^u du \\ &= -\frac{1}{4} e^u + C \\ &= -\frac{1}{4} e^{-2X^2} + C\end{aligned}$$

Check :

$$\begin{aligned}\frac{d}{dY} \left( -\frac{1}{4} e^{-2X^2} + C \right) &= -\frac{1}{4} e^{-2X^2} (-4X) \\ &= X e^{-2X^2}\end{aligned}$$

## Test 23

1. A  $\sec X + C$

2. C  $-\frac{1}{2} \int e^{-2X}(-2) = -\frac{1}{2} e^{-2X} + C$

3. D  $3 \int (X^2 - 2X) dX = \int X^2 dX - 2 \int X dX$   
 $= \frac{X^3}{3} - 2 \frac{X^2}{2} + C$   
 $= \frac{1}{3} X^3 - X^2 + C$

4. C Let  $u = 3 - Y$ ;  $\frac{du}{dY} = -1$ ;  $du = -dY$

$$\int \frac{dY}{(3-Y)^3} = - \int \frac{dY(-1)}{(3-Y)^3}$$

$$= - \int u^{-3} du$$

$$= - \frac{u^{-2}}{-2} + C$$

$$= \frac{1}{2} (3-Y)^{-2} + C$$

5. A Let  $u = 5 - 3X^2$ ;  $\frac{du}{dX} = -6X$ ;  $du = -6XdX$

$$\int X(5-3X^2)^5 dX = -\frac{1}{6} \int -6X(5-3X^2)^5 dX$$

$$= -\frac{1}{6} \int u^5 du$$

$$= -\frac{1}{6} \frac{u^6}{6} + C$$

$$= -\frac{1}{36} u^6 + C$$

$$= -\frac{1}{36} (5-3X^2)^6 + C$$

6. B Let  $u = 2 - \ln X$ ;  $\frac{du}{dX} = -\frac{1}{X}$ ;  $du = -\frac{1}{X} dX$

$$- \int \frac{(2-\ln X)^3}{X} dX(-1) = - \int u^3 du$$

$$= - \frac{u^4}{4} + C$$

$$= - \frac{1}{4} (2-\ln X)^4 + C$$

7. D  $\int (1+e^X)^2 dX = \int (1+2e^X+e^{2X}) dX$   
 $= \int dX + 2 \int e^X dX + \frac{1}{2} \int e^{2X} dX(2)$   
 $= X + 2e^X + \frac{1}{2} e^{2X} + C$

8. D Let  $u = 1 + \cot 2\theta$ ;  $\frac{du}{d\theta} = -\csc^2 2\theta \cdot 2$ ;  
 $du = -2\csc^2 2\theta d\theta$

$$-\frac{1}{2} \int \frac{\csc^2 2\theta d\theta}{\sqrt{1+\cot 2\theta}} (-2) = -\frac{1}{2} \int \frac{du}{\sqrt{u}}$$

$$= -\frac{1}{2} \int u^{-\frac{1}{2}} du$$

$$= -\frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= -\sqrt{u} + C$$

$$= -\sqrt{1+\cot 2\theta} + C$$

9. A Let  $u = X^3 - 15X + 3$ ;  $\frac{du}{dX} = 3X^2 - 15$ ;  
 $du = (3X^2 - 15)dX$

$$\frac{1}{3} \int \frac{(X^2-5)dX}{X^3-15X+3} (3) = \frac{1}{3} \int u^{-1} du$$

$$= \frac{1}{3} \ln(X^3-15X+3) + C$$

10. B Let  $u = 1 - 2\cos 2\theta$ ;  $\frac{du}{d\theta} = -2(-\sin 2\theta) \cdot 2$ ;  
 $du = 4\sin 2\theta d\theta$

$$\frac{1}{4} \int \frac{\sin 2\theta d\theta \cdot 4}{(1-2\cos 2\theta)^3} = \frac{1}{4} \int u^{-3} du$$

$$= \frac{1}{4} \frac{u^{-2}}{-2} + C$$

$$= -\frac{1}{8} (1-2\cos 2\theta)^{-2} + C$$